PARALLEL IMPLEMENTATION OF COUPLED WAVE AND BOTTOM DEPOSIT TRANSPORTATION MODELS TO SIMULATE SURFACE POLLUTION AREAS

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STATEMENT OF THE PROBLEM OF WAVE HYDRODYNAMICS

The initial equations of hydrodynamics of shallow water bodies are:

- the equation of motion (Navier-Stokes):

$$u'_t + uu'_x + vu'_y + wu'_z = -\frac{1}{\rho} p'_x + \left(\mu u'_x\right)'_x + \left(\mu u'_y\right)'_y + \left(\nu u'_z\right)'_z$$
$$v'_t + uv'_x + vv'_y + wv'_z = -\frac{1}{\rho} p'_y + \left(\mu v'_x\right)'_x + \left(\mu v'_y\right)'_y + \left(\nu v'_z\right)'_z$$
$$w'_t + uw'_x + vw'_y + ww'_z = -\frac{1}{\rho} p'_z + \left(\mu w'_x\right)'_x + \left(\mu w'_y\right)'_y + \left(\nu w'_z\right)'_z - g$$

- the equation of continuity in the case of variable density

$$\rho'_t + \left(\rho u\right)'_x + \left(\rho v\right)'_y + \left(\rho w\right)'_z = 0$$

STATEMENT OF THE PROBLEM OF WAVE HYDRODYNAMICS

The system of equations is considered under the following boundary conditions:

- **Input boundary conditions**

$$u(x,y,z,t)=u(t),\quad v(x,y,z,t)=v(t),\quad p'_n(x,y,z,t)=0,\quad V'_n(x,y,z,t)=0$$

- **boundary conditions for bottom surface**

$$\rho\mu(u')_n(x,y,z,t)=-\tau_x(t),\ \rho\mu(v')_n(x,y,z,t)=-\tau_y(t),\ V_n(x,y,z,t)=0,\ p'_n(x,y,z,t)=0$$

- **boundary conditions for free surface**

$$\rho\mu(u')_n(x,y,z,t)=-\tau_x(t),\ \rho\mu(v')_n(x,y,z,t)=-\tau_y(t),$$

$$w(x,y,t)=-\omega-p'_t/\rho g,\ p'_n(x,y,t)=0$$

STATEMENT OF THE PROBLEM OF WAVE HYDRODYNAMICS

- The components of the tangential stress for the bottom surface, taking into account the notation, may be written as follows:

$$\tau_x = \rho_v C_p\left(|V|\right)u|V| \qquad \tau_y = \rho_v C_p\left(|V|\right)v|V|$$

- The approximation considered below makes it possible to build on the basis of the measured velocity pulsations the coefficient of vertical turbulent exchange, inhomogeneous in depth

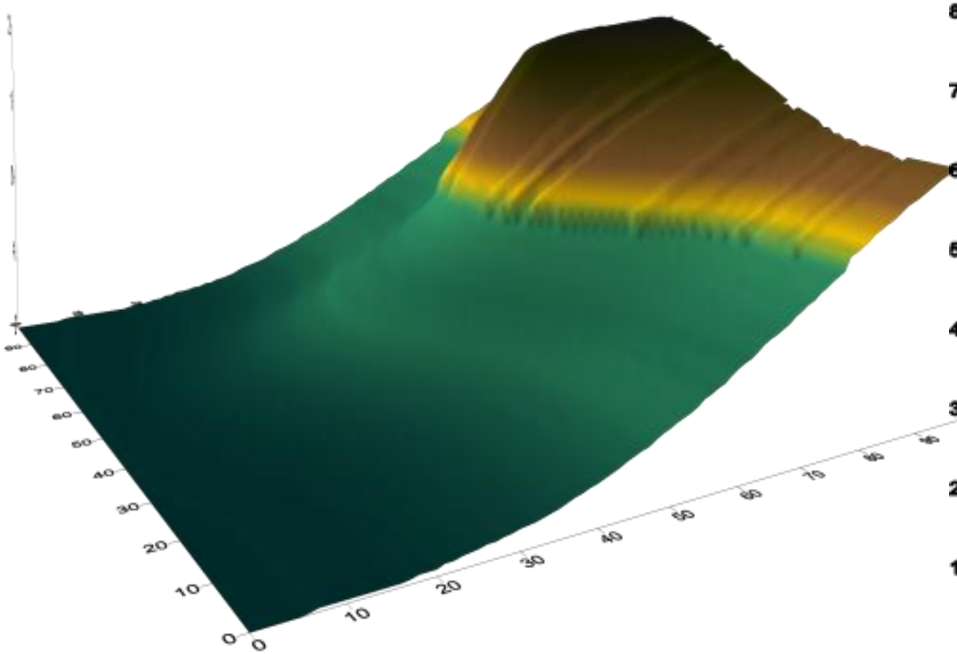
$$\nu = C_s^2 \Delta^2 \frac{1}{2}\sqrt{\left(\frac{\partial \overline{U}}{\partial z}\right)^2 + \left(\frac{\partial \overline{V}}{\partial z}\right)^2}$$

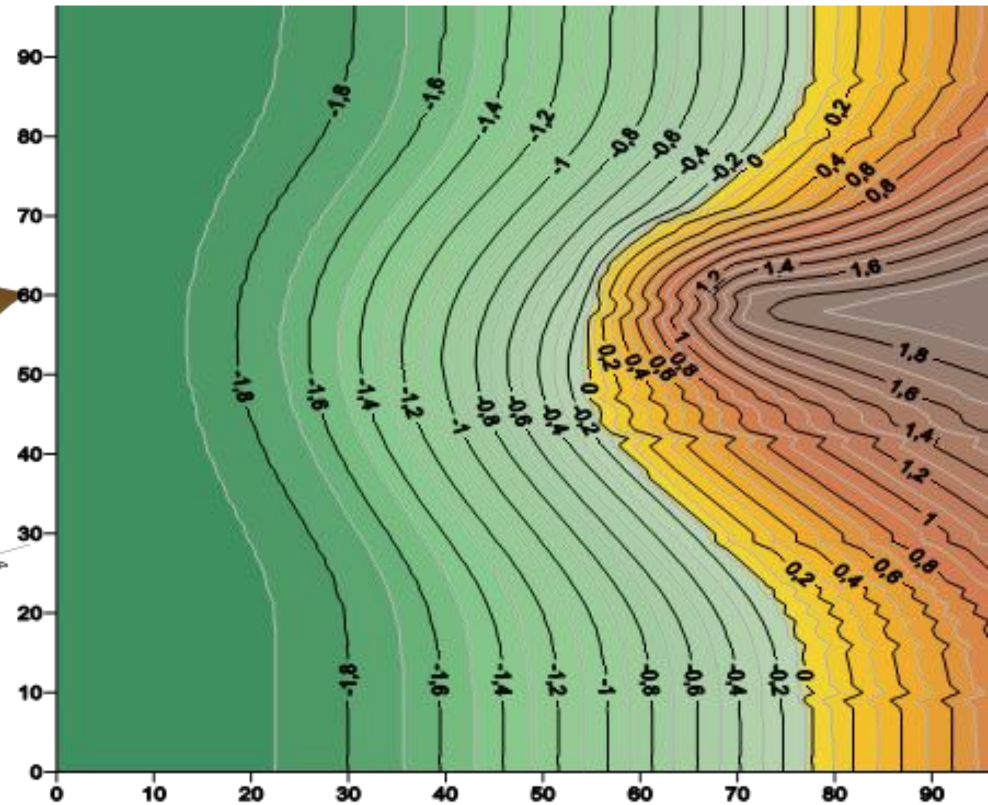
where $\overline{U}, \overline{V}$ are the time-averaged pulsations of the horizontal velocity components

Δ is the characteristic scale of the grid

C_s is Smagorinsky dimensionless empirical constant whose value is usually determined on the basis of calculating the decay process of homogeneous isotropic turbulence

THE GEOMETRY OF THE COMPUTATIONAL DOMAIN





CONTINUOUS 3D MODEL OF DIFFUSION-CONVECTION-AGGREGATION OF SUSPENSIONS

- The system of equations describing the behavior of particles will look like this

$$\begin{cases} \dfrac{\partial c_r}{\partial t}+\dfrac{\partial(uc_r)}{\partial x}+\dfrac{\partial(vc_r)}{\partial y}+\dfrac{a-b}{h}\dfrac{\partial\left(\left(w+w_{g,r}\right)c_r\right)}{\partial\theta}= \\ =\mu\left(\dfrac{\partial^2 c_r}{\partial x^2}+\dfrac{\partial^2 c_r}{\partial y^2}\right)+\left(\dfrac{a-b}{h}\right)^2\dfrac{\partial}{\partial\theta}\left(\nu\dfrac{\partial c_r}{\partial\theta}\right)+F_r, \\ F_1=\left(\alpha_2 c_2-\beta_1 c_1\right)+\Phi_1(x,y,\theta,t), \\ \mathrm{L} \\ F_r=\left(\beta_{r-1}c_{r-1}-\alpha_r c_r\right)+\left(\alpha_{r+1}c_{r+1}-\beta_r c_r\right)+\Phi_r(x,y,\theta,t), \\ \mathrm{L} \\ F_R=\left(\beta_{R-1}c_{R-1}-\alpha_R c_R\right)+\Phi_R(x,y,\theta,t),\ r=2,\ldots,R-1 \end{cases}$$

- Add to the system the initial and boundary conditions:
  - initial conditions at time t=0 $c_1(x,y,\theta,0)\equiv c_{10}(x,y,\theta),\ldots,\ c_r(x,y,\theta,0)\equiv c_{r0}(x,y,\theta),\ldots,$ $c_R(x,y,\theta,0)\equiv c_{R0}(x,y,\theta),\ r=2,\ldots,R-1$
  - boundary conditions on the boundary

$$\frac{\partial c_1}{\partial n}=\ldots=\frac{\partial c_r}{\partial n}=\ldots=\frac{\partial c_R}{\partial n}=0 \qquad \frac{\partial c_1}{\partial n}=-\frac{u_\Gamma}{\mu}c_1,\ldots,\frac{\partial c_r}{\partial n}=-\frac{u_\Gamma}{\mu}c_r,\ldots,\frac{\partial c_R}{\partial n}=-\frac{u_\Gamma}{\mu}c_R$$

  - boundary conditions on the water surface $\dfrac{\partial c_1}{\partial\theta}=\ldots=\dfrac{\partial c_r}{\partial\theta}=\ldots=\dfrac{\partial c_R}{\partial\theta}=0$
  - boundary conditions at the bottom $\dfrac{\partial c_1}{\partial n}=-\dfrac{w_{g,1}}{\nu}c_1,\ldots,\ \dfrac{\partial c_r}{\partial n}=-\dfrac{w_{g,r}}{\nu}c_r,\ \ldots,\dfrac{\partial c_R}{\partial n}=-\dfrac{w_{g,R}}{\nu}c_R.$

Numerical realization of the discrete hydrodynamical model

$$\bar{w}_h = \left\{ t^n = n\tau, x_i = ih_x, y_j = jh_y, z_k = kh_z;\ n = \overline{0..N_t}, i = \overline{0..N_x}, j = \overline{0..N_y}, k = \overline{0..N_z}; \right.$$
$$\left. N_t\tau = T, N_x h_x = l_x, N_y h_y = l_y, N_z h_z = l_z \right\}$$

The method of pressure correction for variable water density :

$$\frac{\tilde{u}-u}{\tau} + u\bar{u}'_x + v\bar{u}'_y + w\bar{u}'_z = \left(\mu\bar{u}'_x\right)'_x + \left(\mu\bar{u}'_y\right)'_y + \left(\nu\bar{u}'_z\right)'_z$$

$$\frac{\tilde{v}-v}{\tau} + u\bar{v}'_x + v\bar{v}'_y + w\bar{v}'_z = \left(\mu\bar{v}'_x\right)'_x + \left(\mu\bar{v}'_y\right)'_y + \left(\nu\bar{v}'_z\right)'_z$$

$$\frac{\tilde{w}-w}{\tau} + u\bar{w}'_x + v\bar{w}'_y + w\bar{w}'_z = \left(\mu\bar{w}'_x\right)'_x + \left(\mu\bar{w}'_y\right)'_y + \left(\nu\bar{w}'_z\right)'_z - g$$

$$p''_{xx} + p''_{yy} + p''_{zz} = \frac{\hat{\rho}-\rho}{\tau^2} + \frac{\left(\hat{\rho}\tilde{u}\right)'_x}{\tau} + \frac{\left(\hat{\rho}\tilde{v}\right)'_y}{\tau} + \frac{\left(\hat{\rho}\tilde{w}\right)'_z}{\tau}$$

$$\frac{\hat{u}-\tilde{u}}{\tau} = -\frac{1}{\rho}\hat{p}'_x \qquad \frac{\hat{v}-\tilde{v}}{\tau} = -\frac{1}{\rho}\hat{p}'_y \qquad \frac{\hat{w}-\tilde{w}}{\tau} = -\frac{1}{\rho}\hat{p}'_z$$

THE DISCRETE MODEL OF HYDRODYNAMICS OF SHALLOW WATER RESERVOIRS

- In the construction of discrete mathematical models of hydrodynamics, the fullness of the control cells was taken into account, which makes it possible to increase the real accuracy of the solution in the case of a complicated geometry of boundary surfaces.
- Through $o_{i,j,k}$ marked «fullness» of the cell (i,j,k) (VOF) .
- The degree of fullness of the cell is determined by the pressure of the liquid column inside this cell. If the average pressure at the nodes that belong to the vertices of the cell in question is greater than the pressure of the liquid column inside the cell, then the cell is considered to be full . In the general case, the «fullness» of the cells can be calculated by the following formula:

$$o_{i,j} = \frac{P_{i,j,k} + P_{i-1,j,k} + P_{i,j-1,k} + P_{i-1,j-1,k}}{4\rho g h_z}$$

THE DISCRETE MODEL OF HYDRODYNAMICS OF SHALLOW WATER RESERVOIRS

In the case of boundary conditions of the third kind

$$c'_n(x,y,t)=\alpha_n c+\beta_n$$

the discrete analogues of the convective uc'_x and diffusion $(\mu c'_x)'_x$ transfer operators, obtained with the help of the variant of finite volume method, taking into account the partial «fullness» of the cells, can be written in the following form:

$$uc'_x \simeq (q_1)_{i,j}\, u_{i+1/2,j}\frac{c_{i+1,j}-c_{i,j}}{2h_x}+(q_2)_{i,j}\, u_{i-1/2,j}\frac{c_{i,j}-c_{i-1,j}}{2h_x},$$

$$(\mu c'_x)'_x \simeq (q_1)_{i,j}\,\mu_{i+1/2,j}\frac{c_{i+1,j}-c_{i,j}}{h_x^2}-(q_2)_{i,j}\,\mu_{i-1/2,j}\frac{c_{i,j}-c_{i-1,j}}{h_x^2}-\left|(q_1)_{i,j}-(q_2)_{i,j}\right|\mu_{i,j}\frac{\alpha_x c_{i,j}+\beta_x}{h_x}.$$

- Similarly, approximations other coordinate directions may be recorded.
- The error in approximating the mathematical model is equal to $\mathrm{O}\left(\tau+\|h\|^2\right)$, where $\|h\|=\sqrt{h_x^2+h_y^2+h_z^2}$.

THE DISCRETE MODEL OF HYDRODYNAMICS OF SHALLOW WATER RESERVOIRS

The conservation of the flow at the discrete level of the developed hydrodynamic model is proved, as well as the absence of non-conservative dissipative terms obtained as a result of discretization of the system of equations. A sufficient condition for the stability and monotony of the developed model is determined on the basis of the grid maximum principle, with constraints on the step with respect to the spatial coordinates:

$$h_x < \left|2\mu / u\right| \qquad h_y < \left|2\mu / v\right| \qquad h_z < \left|2\nu / w\right|$$

or

$$\mathrm{Re} \le 2N$$

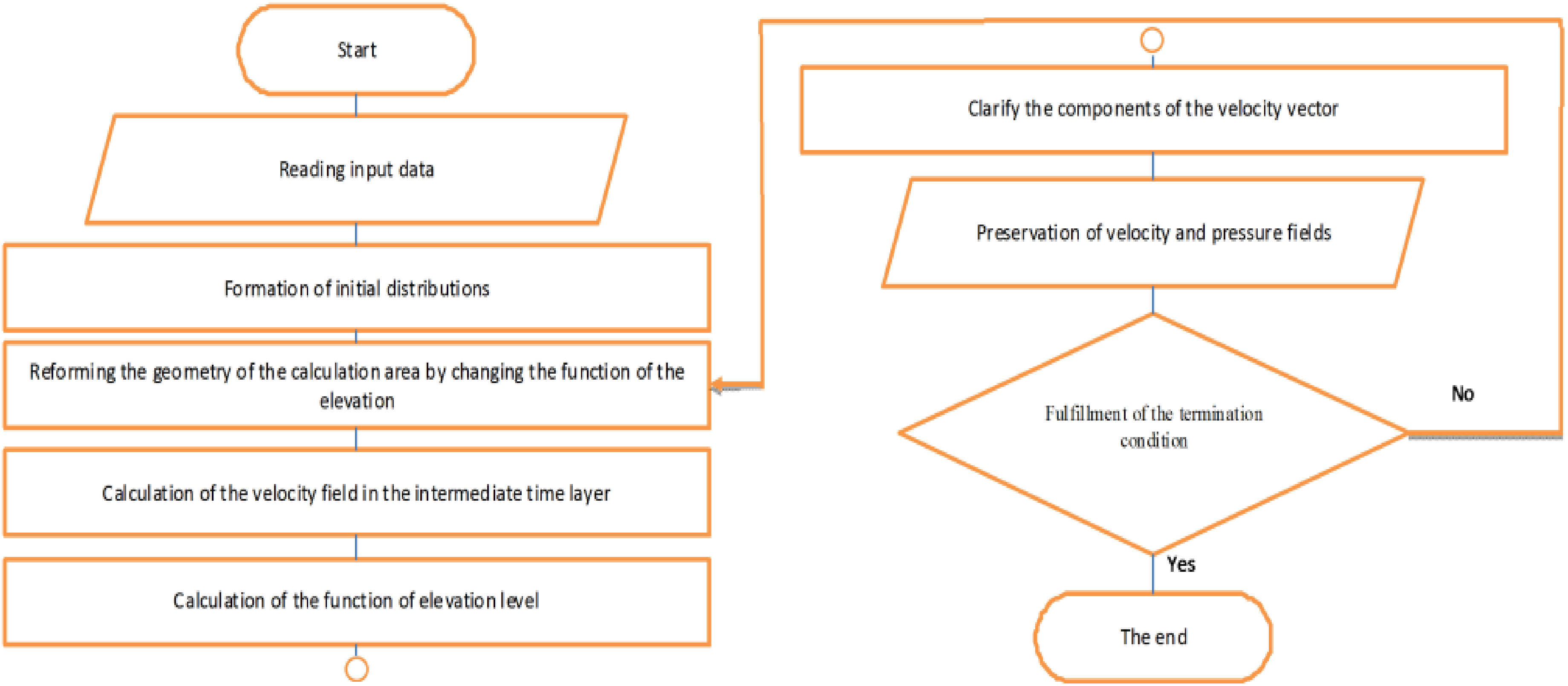
where $\mathrm{Re} = \left|V\right| \cdot l / \mu$ is the Reynolds number,

l is the characteristic size of the domain, $N = \max\left\{N_x, N_y, N_z\right\}$

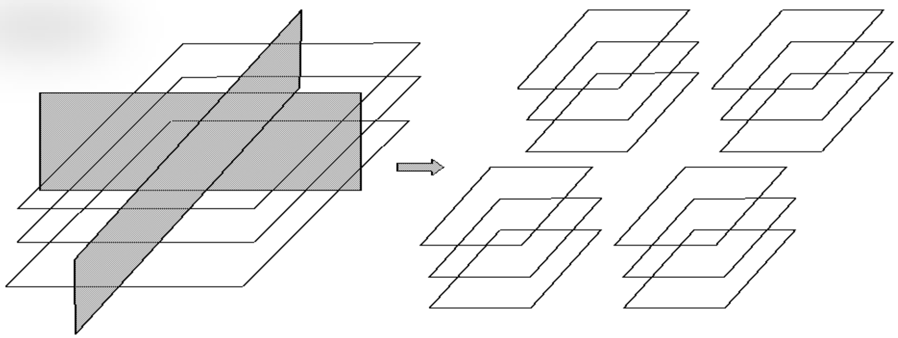
Discrete analogs of the system of equations are solved by an adaptive modified alternating-triangular method of variational type

(the variant of Adaptive Iterative Symmetric Successive Over Relaxation Method, which is development of A.N. Konovalov –A.A. Samarskii methods)

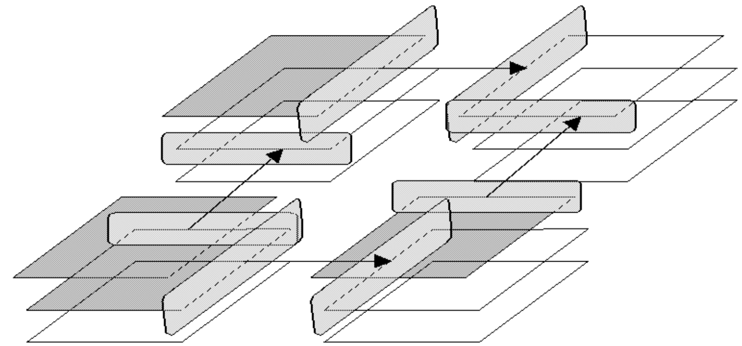
DESCRIPTION OF THE SOFTWARE PACKAGE



PARALLEL VERSION OF THE ALGORITHM FOR SOLVING GRID EQUATIONS



Domain decomposition in two spatial directions



Calculation of the correction vector

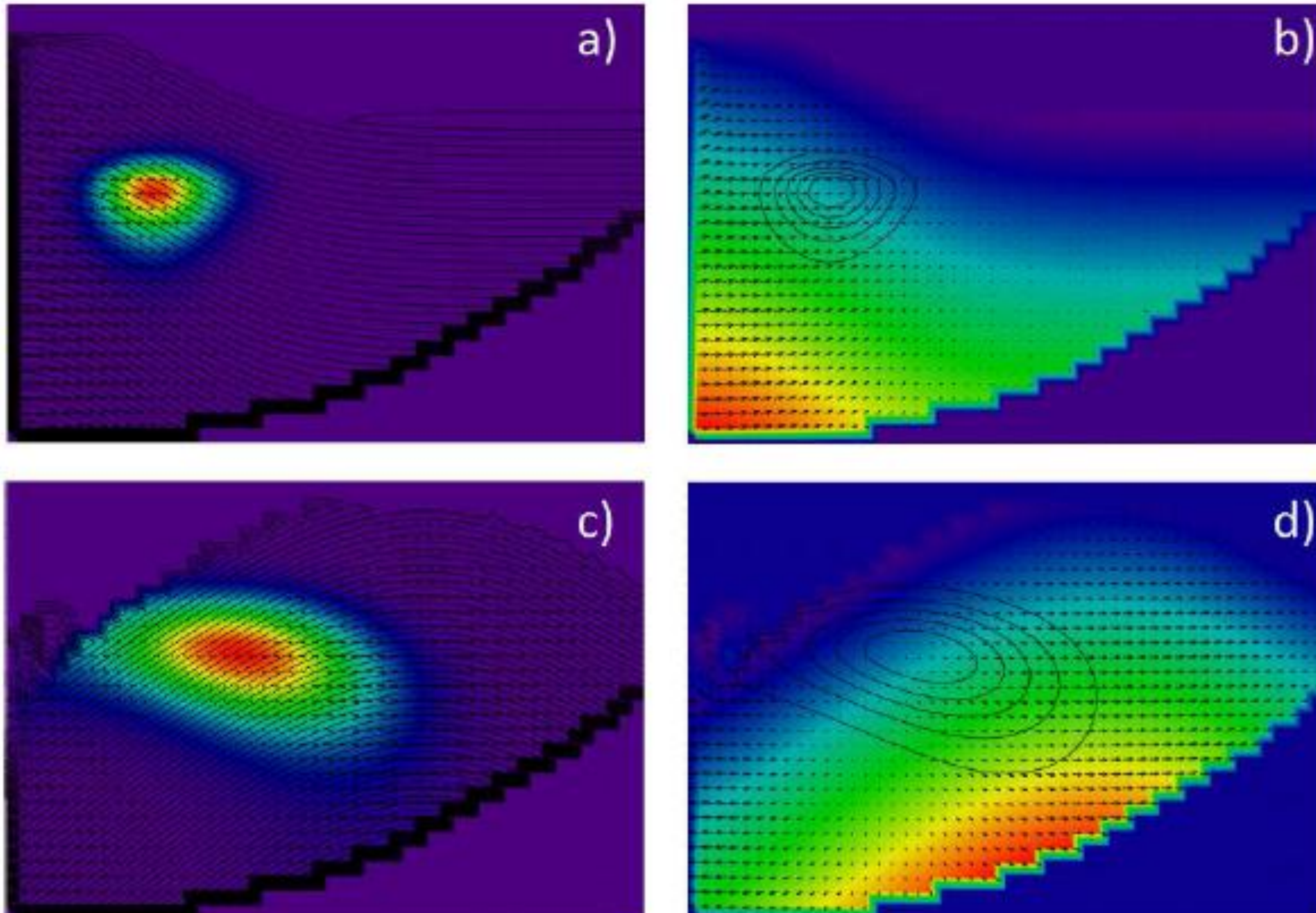
Acceleration and efficiency in dependence of processor numbers

Number of processors	Time, sec.	Acceleration	Efficiency
1	7,490639	1	1
2	4,151767	1,804	0,902
4	2,549591	2,938	0,734
8	1,450203	5,165	0,646
16	0,882420	8,489	0,531
32	0,458085	16,351	0,511
64	0,265781	28,192	0,44
128	0,171535	43,668	0,341

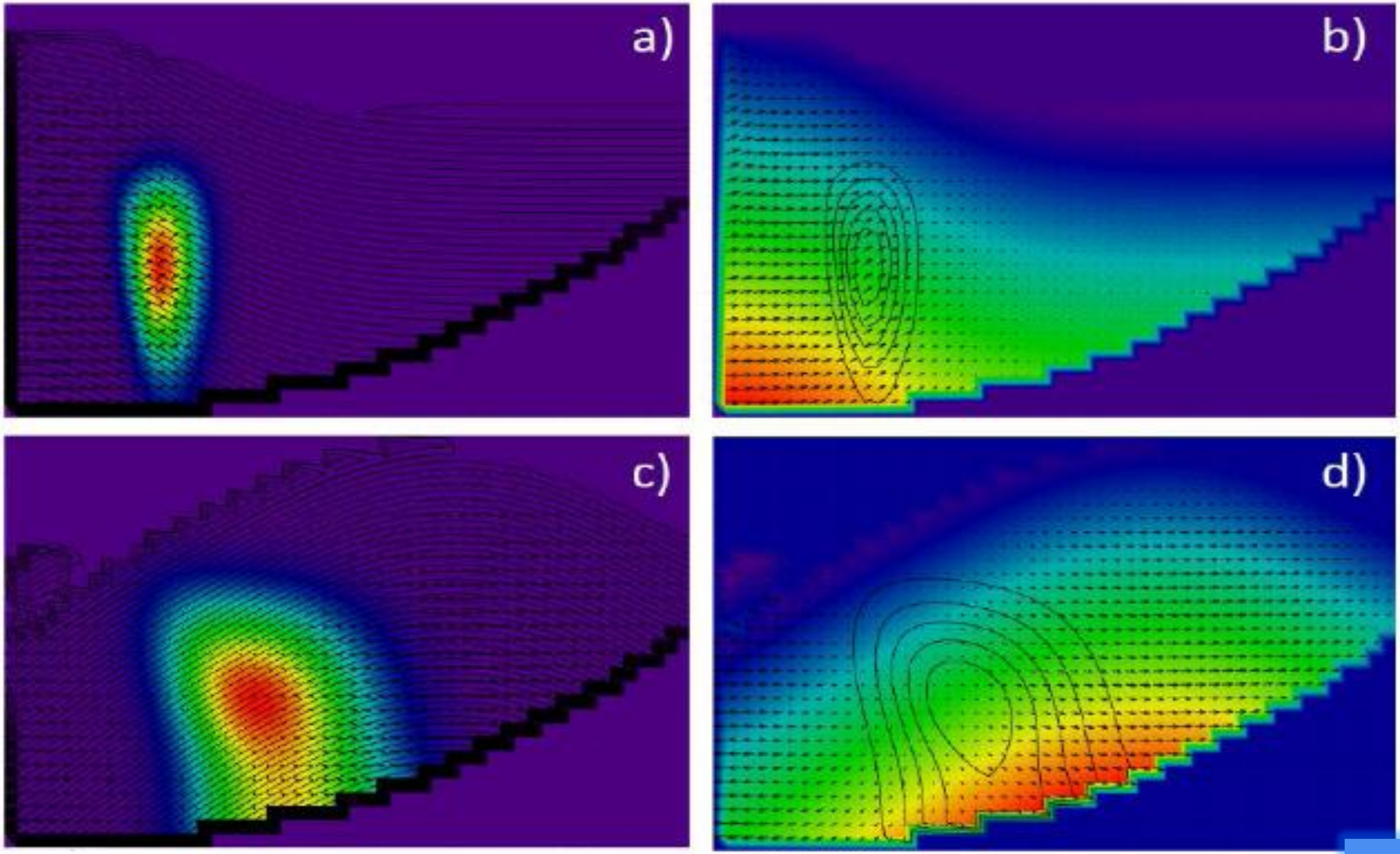
MEASUREMENT OF PARAMETERS OF WAVE PROCESSES ON THE BASIS OF FIELD OBSERVATIONS

№	Depth, cm	Wave length, s	Averagewave height, cm	Maximum value of wave height, cm	Dispersion of level elevation function	Correlation with normal distribution	Correlation with lognormal distribution
1	12.734	3.181	1.434	3.266	3.384	0.67622403	0.72818161
2	21.657	3.187	2.216	5.127	2.875	0.71970734	0.75497854
3	34.296	3.257	2.673	6.673	2.587	0.76756352	0.80809736
4	47.696	3.208	2.903	7.278	2.373	0.80434285	0.81516631
5	50.221	3.238	3.408	8.779	2.465	0.80072646	0.82234947
6	56.95	3.323	3.423	10.05	2.539	0.82520735	0.83499856
7	58.256	3.094	3.538	13.742	2.468	0.70451786	0.75010325
8	75.284	3.482	3.595	12.716	2.317	0.80464887	0.82816629
9	83.353	3.056	4.472	14.647	2.498	0.7677805	0.80442466
10	123.251	3.23	4.671	15.749	2.327	0.78716382	0.82809779

GRAPHS OF ELEVATION AND BOTTOM ELEVATION FUNCTIONS



THE SIMULATION OF THE «JET EFFECT»



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