Simulation of Seismic Waves Propagation in Multiscale Media:
Impact of Cavernous/Fractured Reservoirs

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Abstract. In order to simulate the interaction of seismic waves with cavernous/fractured reservoirs, a finite-difference technique based on locally refined time-and-space grids is used. The need to use these grids is due primarily to the differing scale of heterogeneities in the reference medium and the reservoir. Domain Decomposition methods allow for the separation of the target area into subdomains containing the reference medium (coarse grid) and reservoir (fine grid). Computations for each subdomain can be carried out in parallel. The data exchange between each subdomain within a group is done using MPI through nonblocking iSend/iReceive commands. The data exchange between the two groups is done simultaneously by coupling the coarse and fine grids.
The results of a numerical simulation of a carbonate reservoir are presented and discussed.

Keywords: Finite-difference schemes. Local grid refinement. Domain decomposition. MPI. Group of Processor Units. Master Processor Unit.

1 Introduction and Motivation

One of the key challenges in modern seismic processing is to use the surface and/or borehole data to restore the microstructure of the hydrocarbon reservoir. This microstructure can have a significant impact on oil and gas production. In particular, in many cases the carbonate reservoir's matrix porosity contains the oil but the permeability is mainly through the fracture corridors. In some carbonate reservoirs the in-place oil is contained in karstic caves. Because of this, the ability to locate these microstructures precisely and to characterize their properties is of a great importance. Recently various techniques have been developed to perform this analysis with the help of scattered seismic waves. Among them, the scattering index presented by Willis et al. ([9]) or a variety of the imaging techniques recently developed under the generic name of interferometry (see e.g. book of G.Schuster [7]).

The first step in the development of any inversion/imaging procedure is to simulate accurately the wave field scattered by fractures and caves. The numerical and computer constraints even on very large clusters place limitations on the resolution of the model described. Really, a reservoir beds typically at a depth of 2000 ÷ 4000 meters, which is about 50÷70 dominant wavelength. The current practice for the finite-difference simulation of seismic waves propagation at such distances is to use grid cells of 0.05 - 0.1 of a dominant wavelength, usually between 5 - 10 meters. So, one needs to upscale heterogeneities associated with fracturing on a smaller scale (0.01 - 1 meter) and to transform them to an equivalent/effective medium. This effective medium will help reproduce variations in the travel-times and an average change of reflection coefficients but absolutely cancels the scattered waves that are a subject of the above mentioned methods for characterizing fracture distributions.

Thus, the main challenge with a full scale simulation of cavernous/fractured (carbonate) reservoirs in a realistic environment is that one should take into account both the macro- and microstructures. A straightforward implementation of finite difference techniques provides a highly detailed reference model. From the computational point of view, this means a huge amount of memory required for the simulation and, therefore, extremely high computer cost. In particular, a simulated model of dimension 10km × 10km × 10 km, which is common for seismic explorations, with a cell size of 0.5m claims 8×10^{12} cells and needs in $\approx 350Tb$ of RAM.

The popular approach to overcome these troubles is to refine a grid in space only and there are many publications dealing with its implementation (see [6] for a detailed review), but it has at least two drawbacks:

- To ensure stability of the finite-difference scheme the time step must be very small everywhere in the computational domain;
- Unreasonably small Courant ratio in the area with a coarse spatial grid leads to a noticeable increase in numerical dispersion.

Our solution to this issue is to use a mutually agreed local grid refinement in time and space: spatial and time steps are refined by the same factor.

2 Numerical and Parallel Implementation

In our considerations propagation of seismic waves is simulated with help of an explicit finite-difference scheme (FDS) on staggered grids approximating elastic wave equations (velocity-stress formulation):

$$\varrho \frac{\partial \boldsymbol{u}}{\partial t} - A\frac{\partial \boldsymbol{\sigma}}{\partial x} - B\frac{\partial \boldsymbol{\sigma}}{\partial y} - C\frac{\partial \boldsymbol{\sigma}}{\partial z} = 0;$$
$$D\frac{\partial \boldsymbol{\sigma}}{\partial t} - A^T\frac{\partial \boldsymbol{u}}{\partial x} - B^T\frac{\partial \boldsymbol{u}}{\partial y} - C^T\frac{\partial \boldsymbol{u}}{\partial z} = \boldsymbol{f};$$

written for vectors of the velocity $\boldsymbol{u} = (u_x, u_y, u_z)^T$ and the stress $\boldsymbol{\sigma} = (\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xz}, \sigma_{yz}, \sigma_{xy})$.

Staggered grid finite difference scheme updates values of unknown vectors in two steps:

1. from velocities at t to stresses at $t + \Delta t/2$;
2. from stresses at $t + \Delta t/2$ to velocities at $t + \Delta t$.

In view of the local spatial distribution of the stencil used in this finite difference scheme to update the vector at some point M and time $(t + \Delta t/2)$, the previous time level (t) corresponding values should be known in a neighborhood of this point.

Parallel implementation of this FDS is based on the decomposition of the computational domain to elementary subdomains, being assigned to its individual Processor Unit (PU) (Fig.1). Update unknown vectors while moving from a time layer to the next one requires two adjacent PU to exchange unknown vectors values in the grid nodes along the interface. Necessity of this exchange negatively impacts scalability of the method. However, the impact is less vivsble on 3D Domain Decomposition (DD) than in one- and two-dimensional ones (see theoretical estimates of acceleration for different versions of DD in Fig.2)). In our implementation we choose 3D Domain Decomposition, moreover, in order to reduce the idle time, the asynchronous computations based on nonblocking MPI procedures iSend/iReceive are used.

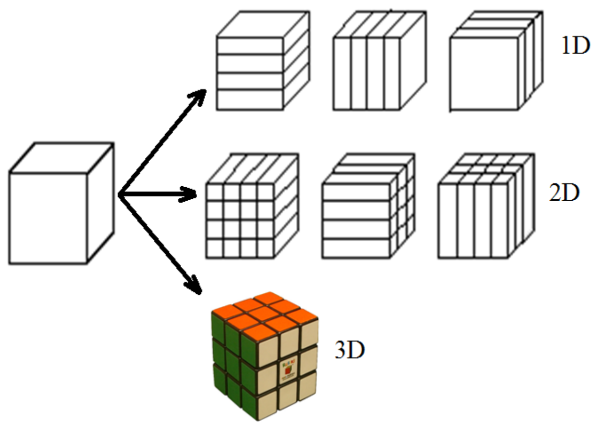

Fig. 1. Domain decomposition. From top to bottom: 1D, 2D, 3D.

In order to carry out the numerical simulation of seismic waves propagation through a multiscale medium we represent it as a superposition of the reference medium given on a coarse grid and the reservoir on a fine grid (see Fig.3). Each of these grids is again decomposed to elementary subdomains being assigned to individual PU. Now these PU are combined into two groups for coarse and fine grids, and special efforts should be applied in order to couple these groups.

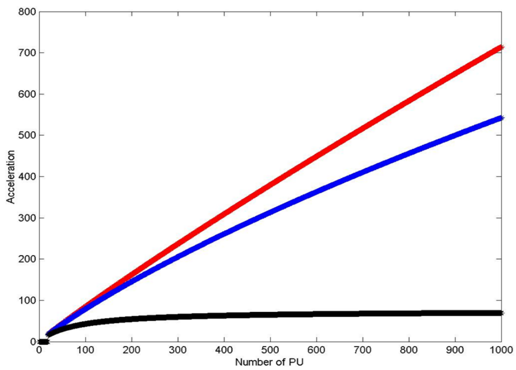

Fig. 2. Theoretical estimation of acceleration for different implementations of Domain Decomposition (top down): 3D, 2D and 1D (see Fig.1).

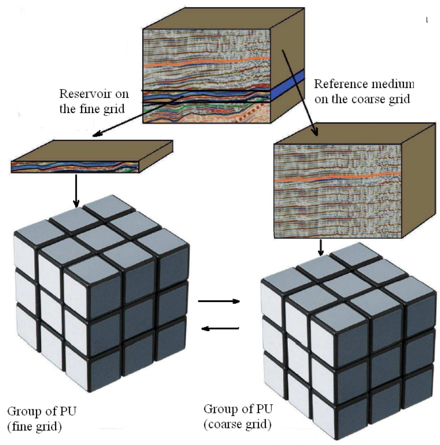

Fig. 3. Two groups of Processor Units.

2.1 Coupling of coarse and fine grids.

First of all, let us explain how a coarse and a fine grids are coupled to each other. The necessary properties of the finite difference method based on a local grid refinement should be its stability and an acceptable level of artificial reflections. Scattered waves we are interested in have an amplitude of about 1% of the incident wave. Artifacts should be at least 10 times less, that is about 0.1% of the incident wave. If we refine the grid at once in time and space stability of the FDS on this way (see [1] and [2]) can be provided via coupling coupling coarse and fine grids on the base of energy conservation, which leads to an unacceptable level (more than 1%) of artificial reflections (see [3], [4]). We modify the approach so that the grid is refined by turn in time and space on two different surfaces surrounding the target area with microstructure. This allows decoupling temporal and spatial grid refinement and to implement them independently and to provide the desired level of artifacts.

Refinement in time. Refinement in time with a fixed 1D spatial discretization is clearly seen in Fig.4 and does not need any explanations. Its modification for 2D and 3D media is straightforward (see [3], [4] for more detail).

Refinement in space. In order to change spatial grids, the Fast Fourier Transform (FFT) based interpolation is used. Let us explain this procedure for a 2D problem. The mutual disposition of a coarse and a fine spatial grids is presented in Fig.4b, which corresponds to updating the stresses by velocities (updating stresses by velocities is implemented in the same manner). As can be seen, to update the stresses on a fine grid it is necessary to know the displacement at the points marked with small (red) triangles, which do not exist on the given coarse grid. Using the fact that all of them are on the same line (on the same plane for 3D statement), we seek the values of missing nodes by FFT based interpolation. Its main advantages are an extremely high performance and exponential accuracy. It is this accuracy allows us to provide the required low level of artifacts (about 0.001 with respect to the incident wave) generated on the interface of these two grids. For 3D statement we again perform the FFT based interpolation but this time 2D.

2.2 Implementation of parallel computations.

Our objective is to analyze the impact of cavernous-fractured reservoirs in the seismic waves for realistic 3D heterogeneous media. Therefore, parallel computations are necessary both in the reference medium, described by a coarse mesh, and in the reservoir itself, determined on a fine grid. The simultaneous use of a coarse and a fine grids and the need for interaction between them makes it difficult to ensure a uniform load of Processor Units under parallelization of computations based on Domain Decomposition. Besides, the user should be allowed to locate the reservoir anywhere in the background.

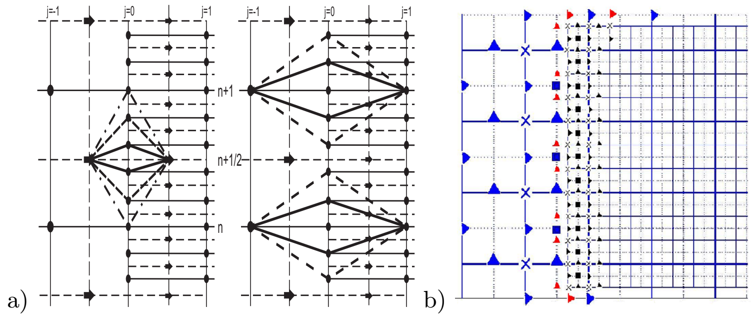

Fig. 4. From a coarse to a fine grid: a) refinement in time (left - displacement, right - stresses) b) refinement in space.

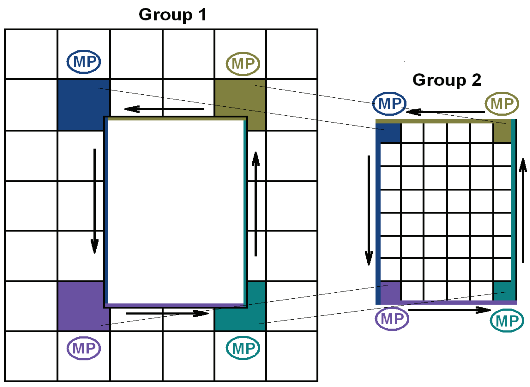

Fig. 5. Processor Units for a coarse (left) and a fine (right) grids. Relevant MP from different groups have the same color.

This problem is resolved through the implementation of parallel computations on two groups of Processor Units. One of them is fully placed 3D heterogeneous referent environment on a coarse grid, while the fine mesh describing the reservoir is distributed among the PU in the second group (Fig.3). Thus, there is a need for both exchanges between processors within each group and between the groups as well. The data exchange within a group is done via faces of the adjacent Processor Units by non-blocking iSend/iReceive MPI procedures. Interaction between the groups is much more complicated. It is carried out not so much for data sending/receiving only, but for coupling a coarse and a fine grids as well. Let us consider the data exchange from the first group (a coarse grid) of PU to the second (a fine grid) and backwards.

From coarse to fine. First are found Processor Units in the first group which cover the target area, and are grouped along each of the faces being in contact with the fine grid. At each of the faces there is allocated the Master Processor (MP), which gathers the computed current values of stresses/displacements and sends them to the relevant MP on a fine grid (see Fig.5). All the subsequent

data processing providing the coupling of a coarse and a fine grids by the FFT based interpolation is performed by the relevant Master Processor in the second group (a fine grid). Later this MP sends interpolated data to each processor in its subgroup.
Interpolation performed by the MP of the second group essentially decreases the amount of sent/received data and, hence, the idle time of PU.
From fine to coarse As in the previous case, primarily there are identified PU from the second group which perform computations on the faces covering the target area. Next, again for each face Master Processor is identified. This MP as its partner from the coarse grid collects data from the relevant face and performs their preprocessing before sending to the first group of PU (a coarse grid). Now we do not need all data in order to move to the next time, but only those of them which fit the coarse grid. Formally, these data could be thinned out, but our experiments have proved that this way generates significant artifacts due to the loss of smoothness. Therefore for this direction (from fine to coarse) we also use the FFT based interpolation implemented by the relevant MP of the second group (a fine grid). The data obtained are sent to the first group.

3 Reservoir Simulation

3.1 2D statement: karstic layer

In order to estimate the accuracy of the method, we first consider a 2D statement for a thin layered reservoir with karst intrusions presented in Fig.6a). In order to describe the microstructure of karstic intrusions we should use a grid with $h_x = h_z = 0.5\,m$, while for the reference medium the dispersion analysis gives $h_x = h_z = 2.5\,m$. In Fig.6a one can see an area with the fivefold grid refinement in time and space. Let us compare now the results of simulation for a uniform fine grid and a grid with the local refinement in time and space. In Fig.6b, one can see a free surface seismogram (horizontal displacement) generated by the vertical point force with a Ricker pulse of a dominant frequency 25 Hz and simulated on the uniform fine grid. Fig.7 represents a comparison of synthetic traces computed on a uniform fine grid and a grid with local refinement in time and space. As can be seen, there is an excellent coincidence of scattered PP-waves and rather good agreement of PS ones.

3.2 3D statement: fracture corridors

Now we present the results of numerical simulation for some realistic model of a carbonate reservoir with fracture corridors. The reservoir is embedded into a homogeneous background with elastic properties equivalent to an average carbonate rock:

$$V_p = 4500m/s,\ V_s = 2500m/s,\ \text{density}\ \varrho = 2500kg/m^3$$

The reservoir is treated as a horizontal layer 200m thick and corresponds to a slightly softer rock with the elastic waves propagation velocities $V_p = 4400m/s$,

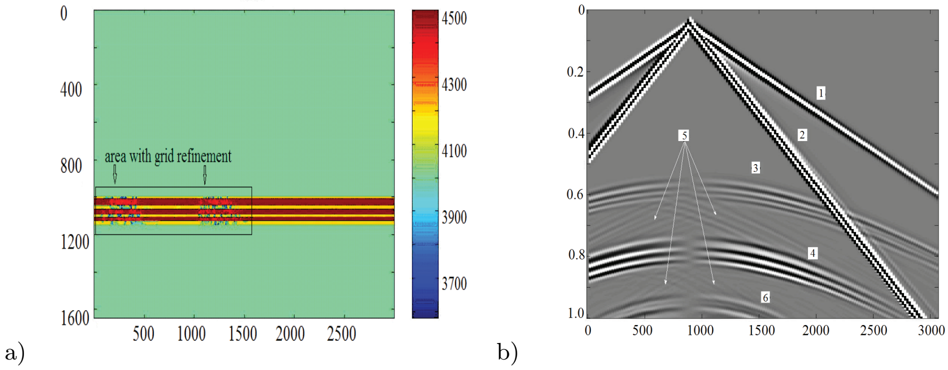

Fig. 6. a) Karstic layer b) Surface seismogram (horizontal component). 1 - direct P-wave, 2 - direct S-wave coupled with surface Rayleigh wave, 3 and 4 - reflected PP- and PS-waves, 5 - scattered PP- and PS-waves, 6 - reflected SP-wave.

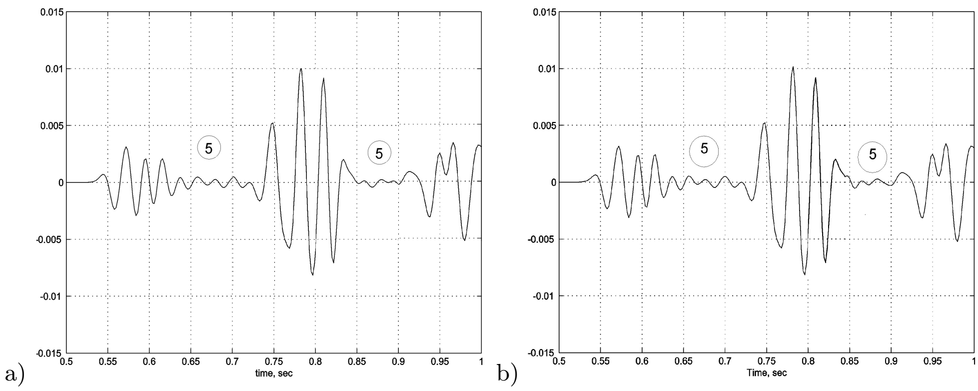

Fig. 7. Traces computed on a uniform fine grid (a) and on a grid with local refinement in time and space.

$V_s = 2400m/s$ and the density $\varrho = 2200kg/m^3$ and contains two fractured layers 30m thick each. The fracturation is of a corridor type, that is, we have included into each layer a set of randomly distributed parallel fracture corridors. The fracture density varies from 0 in the non-fractured facies to 0.3 as a maximum. Finally, the fracture density was transformed to elastic parameters using the second order Hudson theory following [5]. Since fractures were filled with gas, the velocity diminishes down to 3600m/s the lowermost as compared to 4400m/s in the matrix. The fracture corridors were then randomly distributed into fractured layers until the desired fracture density was obtained. The final distribution of fracture corridors can be seen in Fig.8 (two side views).

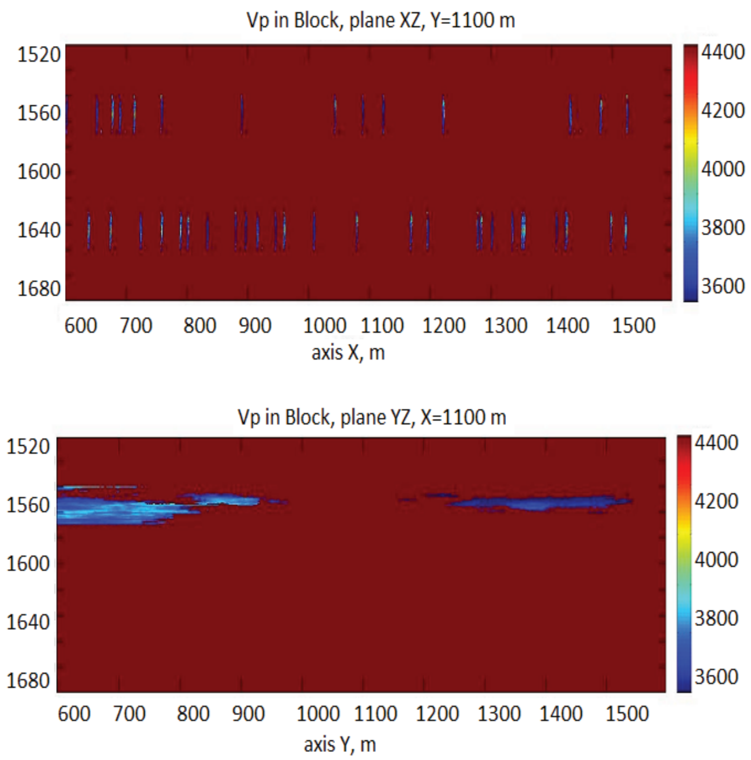

Fig. 8. Side view of fracture corridors within reservoir: orthogonal (top) and parallel (bottom) to the corridor direction

3.3 Synthetic seismograms

The developed parallel software was used for simulation of scattered waves for the reservoir model introduced in the previous section. The acquisition system can be seen in Fig.9. Three-component seismograms are presented in Fig.10. There is a visible difference between the seismograms along the parallel and the perpendicular lines with respect to fracture corridors.

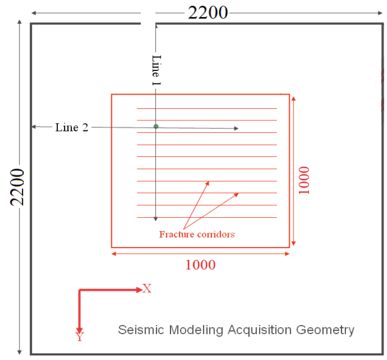

Fig. 9. Acquisition system. The source is at the intersection of Line 1 and Line 2.

4 Conclusion

A finite difference method based on the use of grids with local space-time refinement is proposed, developed and verified. Implementing its parallel software opens up a fundamentally new opportunity to study the processes of formation and propagation of waves scattered by a microstructure of the cavernous/fractured reservoir for a realistic geological environment. The very first simulations carried out using this software, allow the following conclusions:

- Modeling techniques make possible to simulate the impact of fine-scale heterogeneities within a realistic 3D environment in an accurate manner;
- Scattered waves have a significant energy and can be acquired by the field observations, hence there should be a possibility not only to reveal cavities and fractures in the reservoir but to predict their orientation as well.

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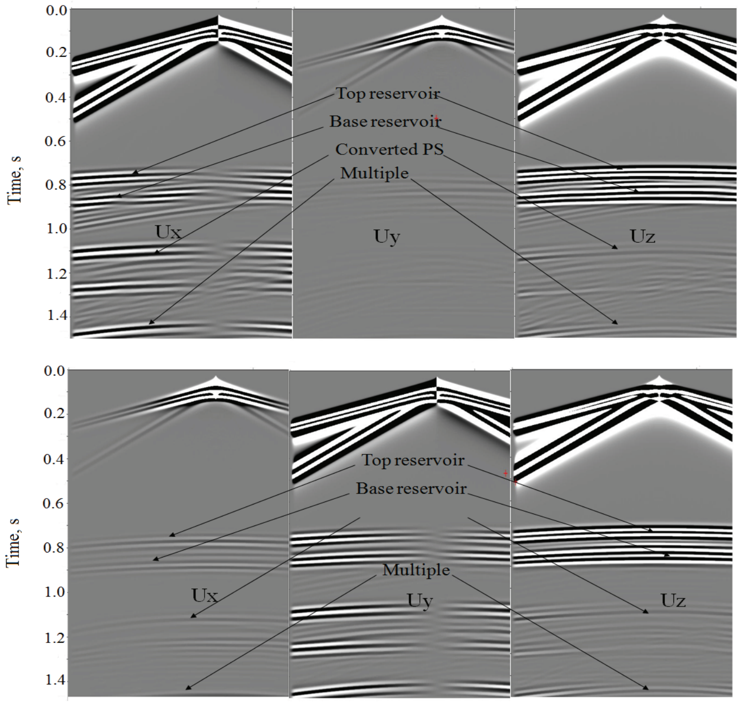

Fig. 10. 3C seismograms along Line 2 (top) and Line 1 (bottom). From left to right: X, Y and Z-displacements.

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