Dynamic optimization of linear solver parameters in modelling of unsteady filtration processes

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Linear systems and linear solvers

- Solution of linear systems
- Linear systems are differ
- A lot of linear solvers
- A lot of parameters for each linear solver
- How to choose the linear solver
- How to choose the linear solver parameters
Goal

- Time optimization of $A_k x = b_k$
- INMOST
- BiILU2 linear solver
- $\tau$ - threshold parameter
- $q$ - overlap parameter
Properties

\[ T_k = \phi(A_k, b_k, p, \varepsilon_k) \equiv \Phi(A_k, b_k, p) \pm \varepsilon_k \equiv \text{Time}(A_k x = b_k) \]

- \( T_k \) may differ even for the same \( A_k, b_k, p \)
- The value of \( \varepsilon_k \) is impossible to predict
- \( T_k \) can be calculated only once
- Optimal value of \( p \) may depend on \( k \)
- \( \min T_k \) may increase during simulation
Several approaches

- Prescribed optimal fixed parameters
- Brute-force search (direct search over all set of parameters)
- Fast simulated re-annealing optimization algorithm
- Alternating parameters probe based tuning (1U)
Fast simulated re-annealing

- Effective random search
- Local terms: “temperature”, “energy”, probability of jump
- Previous values of $T_k$ are not required
- Does not “stop” at the local minima
Alternating parameters probe based tuning (1U)

- Find local minima
- Check nearby area
- Move to global minima
Models

- **Steady**
  - $A_k \equiv A$ and $b_k \equiv b$
  - Research the impact of the parameter $p$ on solution time

- **Model function**
  - Easy to debug and to test
  - Optimal parameter $p$ depends on “time“

- **Unsteady**
  - Black-Oil simulator BOSS
  - Optimal parameter $p$ depends on “time“
  - $min T_k$ depends on simulation step $k$
INM cluster configuration

- Compute Node Arbyte Alkazar+ R2Q50;
- 16 cores (two 8-core processors Intel Xeon E5-2665@2.40GHz);
- 64 Gb RAM;
- SUSE Linux Enterprise Server 11 SP1 (x86_64).
N14 sample problem

Figure: The porosity and permeability distributions for SPE-10 problem

Black-Oil Simulator for Scholars (BOSS) for SPE-10 problem. The size of the model mesh is $60 \times 220 \times 85$ cells ($1.122 \cdot 10^6$ cells). The porosity varies from $1.3 \cdot 10^{-5}$ to 0.5 (see Fig. left). The permeability varies from $10^{-3}$ to $3 \cdot 10^4$ (see Fig. right). The model has 5 vertical wells completed throughout formation. The dimension of the linear system N14 is 3,896,013 unknowns.
Figure: Total solution time $T$ in s. for N14 depending on $\tau$ and $q$ for $p = 16$

$T = f(q = 3, \tau)$ and $T = f(q, \tau = 0.003)$, respectively.
Figure: Total solution time $T$ in s. for N14 in variables $\tau$ and $q$ for $p = 16$
$f(\tau, q) - \text{special function instead of real solution time}$

$$f(\tau, q) = \left( \frac{16}{25} (\lg(\tau/\tau_0))^2 + 1 \right) \left( \frac{1}{25} \left( \frac{17.5(q-q_0)}{7.5 + q - q_0} \right)^2 + 1 \right)$$

$\tau_0 = 0.003, \quad q_0 = 3$

**Figure:** Two-parameter function $f(\tau, q)$
$$f(\tau, q)$$

Figure: Cross-sections for $q = q_{opt} = 3$ and $\tau = \tau_{opt} = 0.003$

$$T = f(q = 3, \tau)$$ and $$T = f(q, \tau = 0.003),$$ respectively.
\[ f(\tau, q, t) - \text{unsteady} \]

\[ \tau_0 = 10^{-2} \cos(2\pi t/t_0) \]

\[ q_0 = 2 + \cos(2\pi t/t_0) \]

\[ t_0 = 100 \]

Here, \( \ln(\tau) \in [-3; -1] \) and \( q \in [1; 3] \).
$f(\tau, q, t) – \text{unsteady}$

Figure: $\tau_{\text{opt}}$ depending on the time step $k$ for function $f(\tau, q, t)$

Brute-force search vrs. SA algorithm and 1U algorithm, respectively.
Unsteady black-oil simulation – fixed parameters \((\tau, q)\)

Figure: Unsteady black-oil simulation solution times depending on time step \(k\)

Figure: Unsteady black-oil simulation cumulative times depending on time step \(k\)
Figure: Optimizing $\tau$ for black-oil simulator
Unsteady black-oil simulation – parameters optimization

**Figure**: Local and cumulative times depending on time step $k$
Unsteady black-oil simulation – parameters optimization

Figure: Cumulative times bar chart for default sets of parameters and for proposed algorithms compared with the optimal one
Conclusion

- Two parameters optimization algorithms are proposed
- The solution time is close (≈10%) to the optimal one
- Better than any prescribed set of parameters
  - 2-3-4 times better than regular set
  - 1.5 times better than the best fixed one

- To be applied to:
  - another linear solvers like PETSc AS\((q)\)+ILU\((k)\)
  - another application, i.e. haemodynamics