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**ПАРАЛЛЕЛЬНЫЙ АЛГОРИТМ  
РАЗВЕРТЫВАНИЯ ФАЗЫ  
ДЛЯ ОБРАБОТКИ ДАННЫХ  
ДИСТАНЦИОННОГО  
ЗОНДИРОВАНИЯ ЗЕМЛИ**

❖ Е Н АИМОВА

❖ В Е МИЛОВ

❖ А В СОСОВИЙ

# OUTLINE

1. Introduction

2. Problem Statement

3. Inverse vortex field method for phase unwrapping

4. Parallel Implementation

5. Numerical experiments

6. Conclusion



# INTRODUCTION

The problem of interferometric phase unwrapping in radar remote sensing of Earth systems is considered. Such interferograms are widely used in the problems of creating and updating maps of the relief of the Earth's surface in geodesy, cartography, environmental monitoring, geological, hydrological and glaciological studies, and for monitoring transport communications.

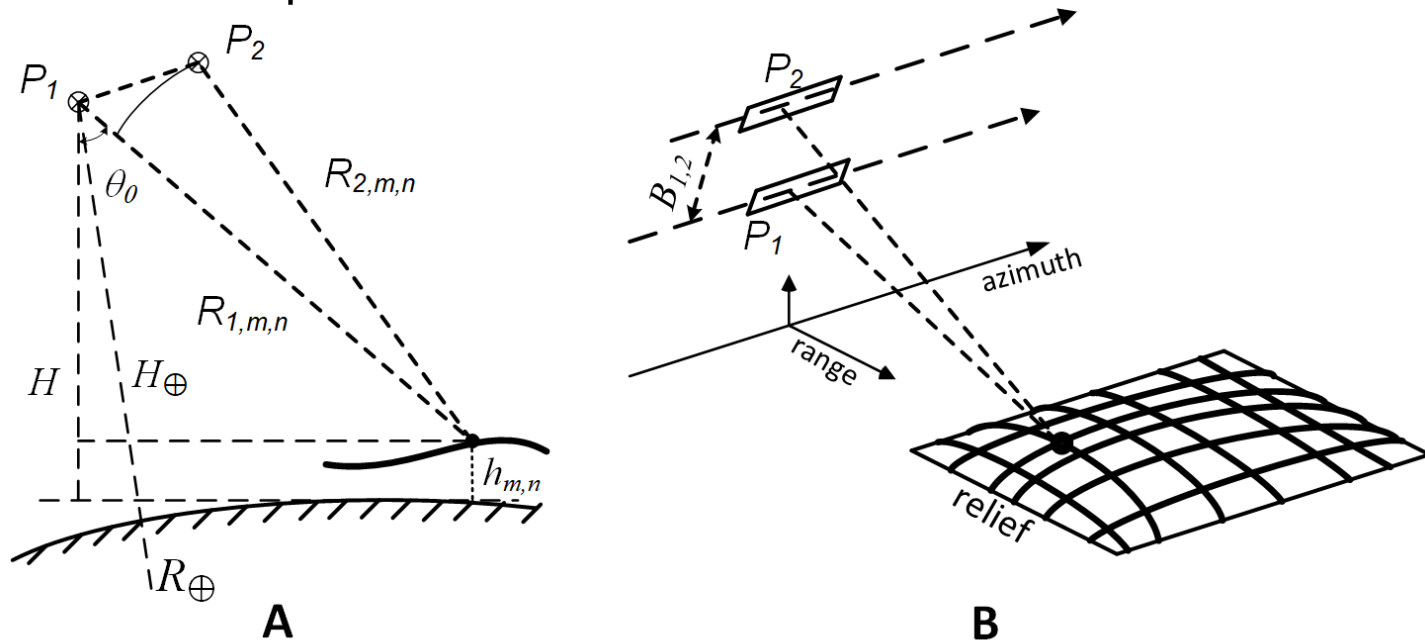
Modern radar systems have ultra-high spatial resolution and a wide band, which leads to the need to unwrap large interferograms from several tens of millions of elements. The implementation of calculations by these methods takes several days.

To solve the phase unwrapping problem, the efficient inverse vortex field method was previously proposed. In this work, we implemented the inverse vortex field method as a parallel code for central processors using the OpenMP and vectorization technologies and for graphics processors using CUDA technology.

Using real radar data, numerical experiments are carried out to study the performance of the developed code. A sufficient accuracy was achieved for a set of images.

## 4 GEOMETRY OF INSAR SURVEY

The initial data for the interferometric survey of the Earth's surface are two complex radar images obtained by a synthesized aperture radar from two parallel orbits located at a short distance.



Here,  $P_1$  and  $P_2$  are positions of the X-ray carrier (centers of synthesized apertures) during observations of the surface element,  $R_1$  and  $R_2$  are the slant ranges,  $R_\oplus$  is the Earth radius,  $B_{1,2}$  is the interferometric baseline,  $\theta_0$  is the incidence angle of the antenna beam,  $H$  is the height of the carrier orbit above the Earth's surface,  $h_{m,n}$  is the height reduced to the geometry of the "flat Earth" for the first

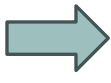
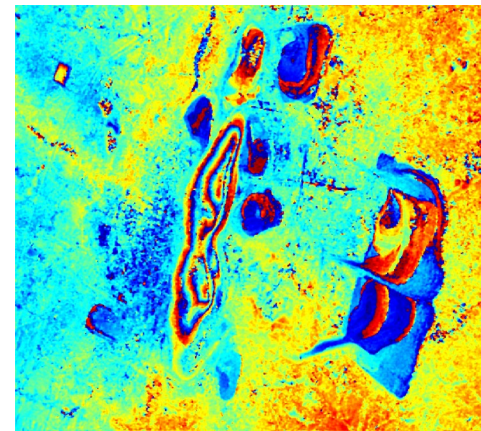
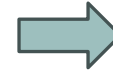
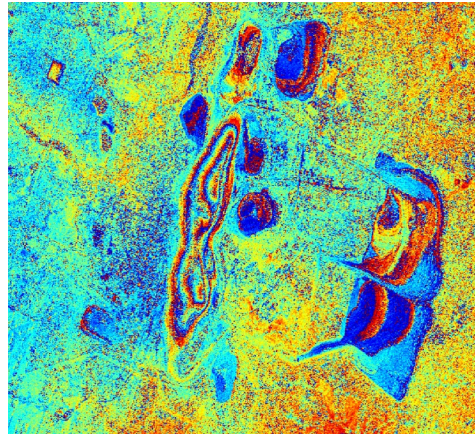
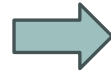
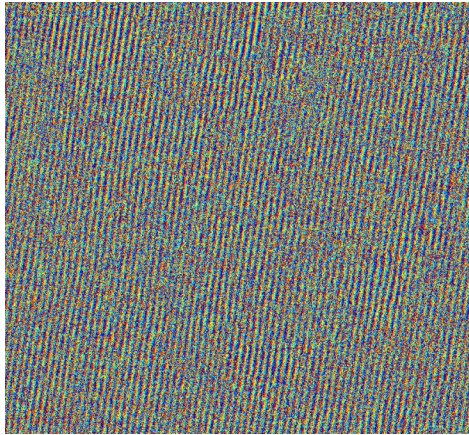
5

# STAGES OF IMAGE PROCESSING

Interferogram  
calculation

Phase noise  
filtration

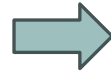
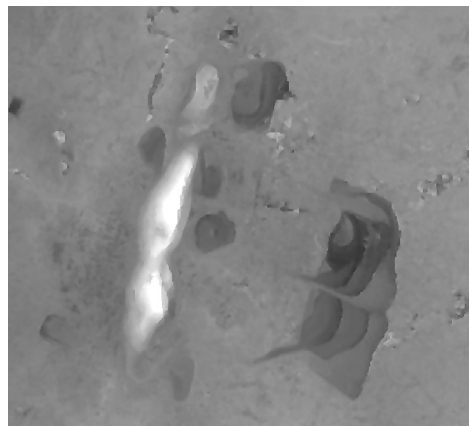
Phase  
unwrapping



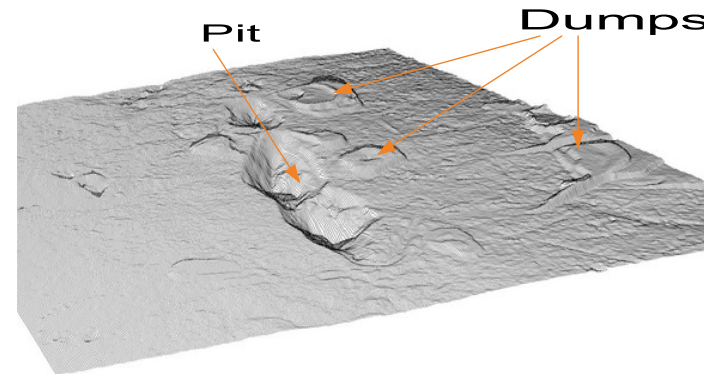
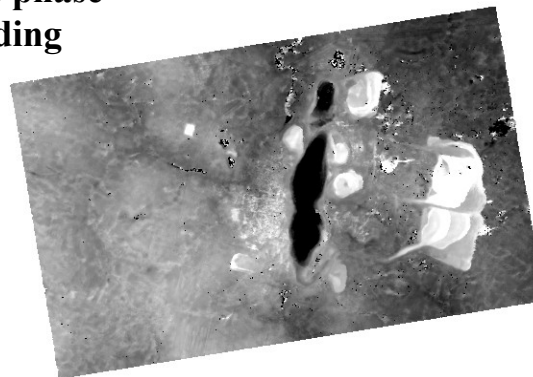
SAR Images

Interferogram

Smoothed interferogram



Absolute phase  
geocoding



Absolute  
(unwrapped) phase

Georeferenced absolute phase and digital elevation model

6

## PROBLEM STATEMENT

The main problem of interferometric radar remote sensing of the Earth from space data processing is phase unwrapping procedure. Phase unwrapping is a process of transforming a two-dimensional relative phase pattern, which takes values only in the interval  $[-\pi, \pi]$ , into an absolute phase.

Phase unwrapping is still the most problematic stage of interferometric processing, since such a problem does not have an analytical solution and is solved approximately.

Processing of interferograms takes a long time and can reach several days, which makes it difficult to use the data for the operative solution of monitoring tasks. To unwrap the phase, algorithms have been developed, many of which have high computational complexity and require the use of approximations and simplifications that lead to a decrease in accuracy.

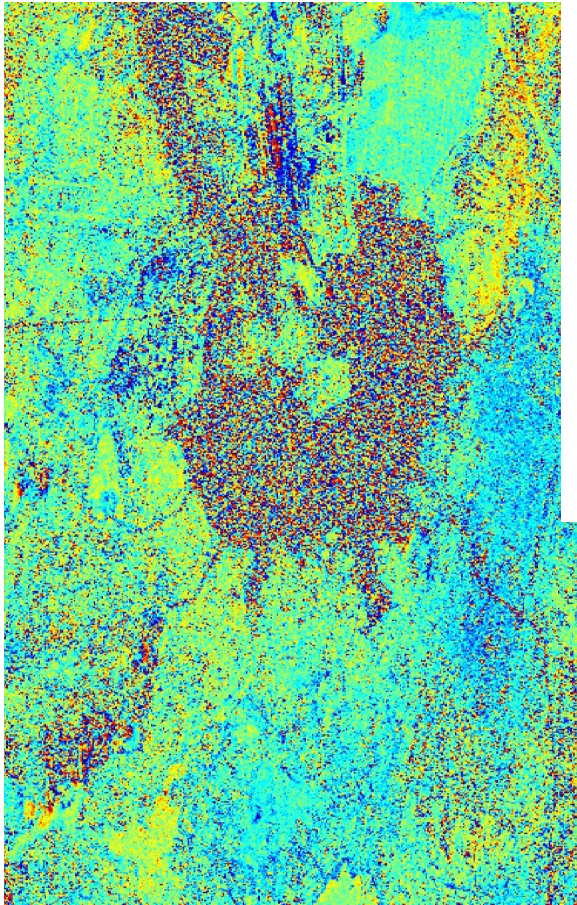
The main problem of phase unwrapping is the presence of phase discontinuities on interferograms.

# 7 INTERFEROMETRIC PHASE DISCONTINUITIES

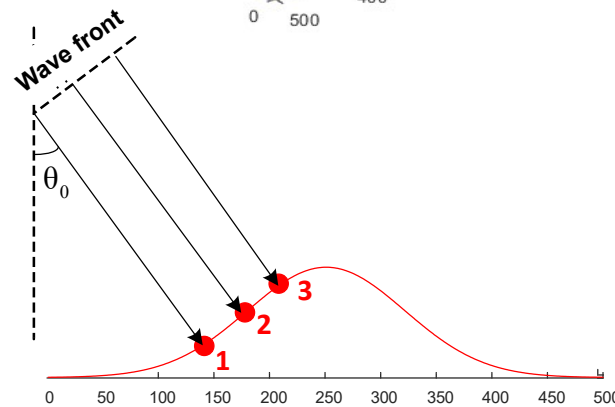
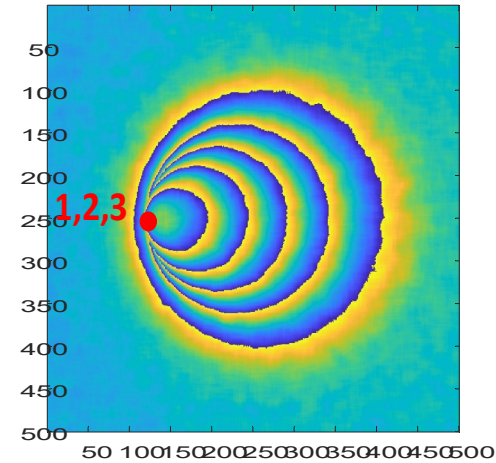
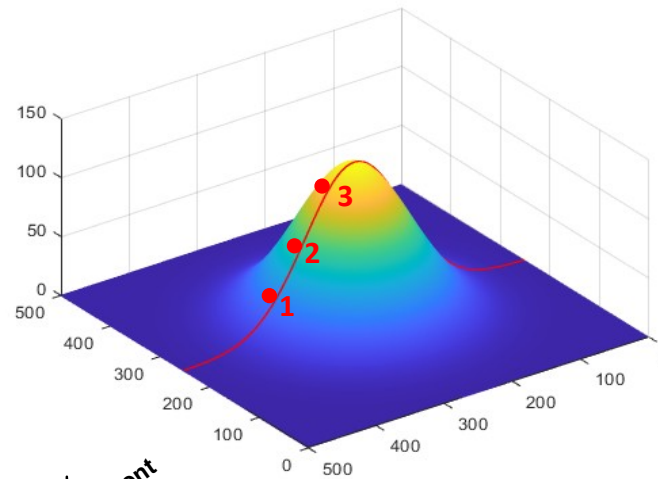
Phase discontinuities appear due to phase noise and the layover effect.

Phase noise appears on the water and forest areas, and the layover effect appears at the areas of the relief gradients

## Phase noise



## Layover



# 8 MATHEMATICAL MODEL OF THE INTERFEROMETRIC PHASE DISCONTINUITIES

Mathematical model of the interferometric phase discontinuities is represented as a complex valued exponential function with a unit amplitude. The argument of exponent is a fractional polynomial function, which roots localize the coordinates of discontinuities.

$$\text{Interferogram } \Delta\varphi_{m,n} = \arg \left\{ \dot{Z}_{1,m,n} \dot{Z}_{2,m,n}^* \right\}$$

Interferometric phase model as a complex value function with a unit amplitude

$$\Delta\varphi_{m,n} \rightarrow \dot{I}(z) = e^{j\Delta\varphi_{m,n}} \Big|_{z=m+jn}$$

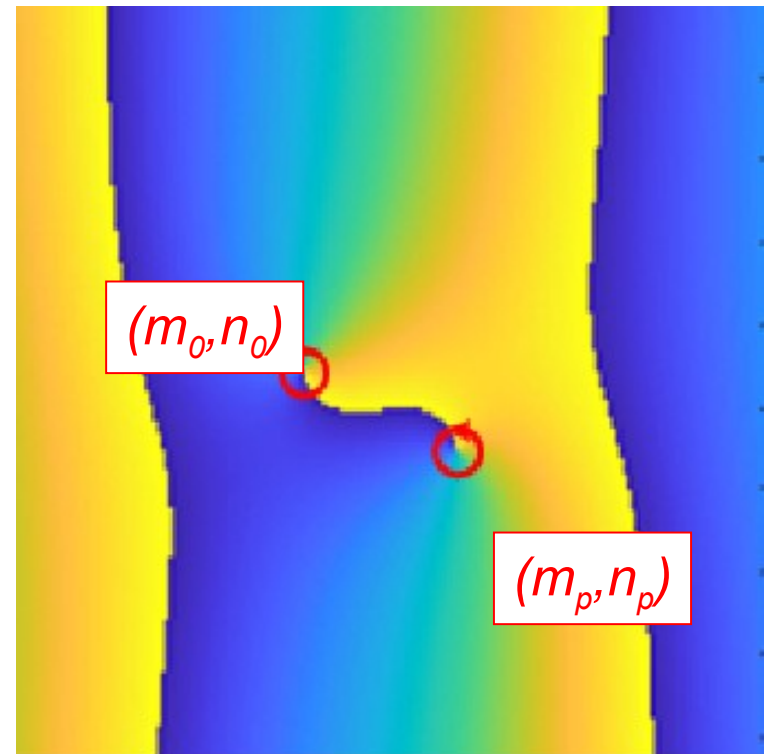
Model of discontinuity point (unit vortex)

$$J^\pm(z) = e^{\pm jq \arg\{z - z_{(0/p)}\}}$$

Model of discontinuity

$$\dot{I}_{\Pi}(z) = \exp \left[ j \arg \left\{ \frac{z - z_{01}}{z - z_{p1}} \right\} \right]$$

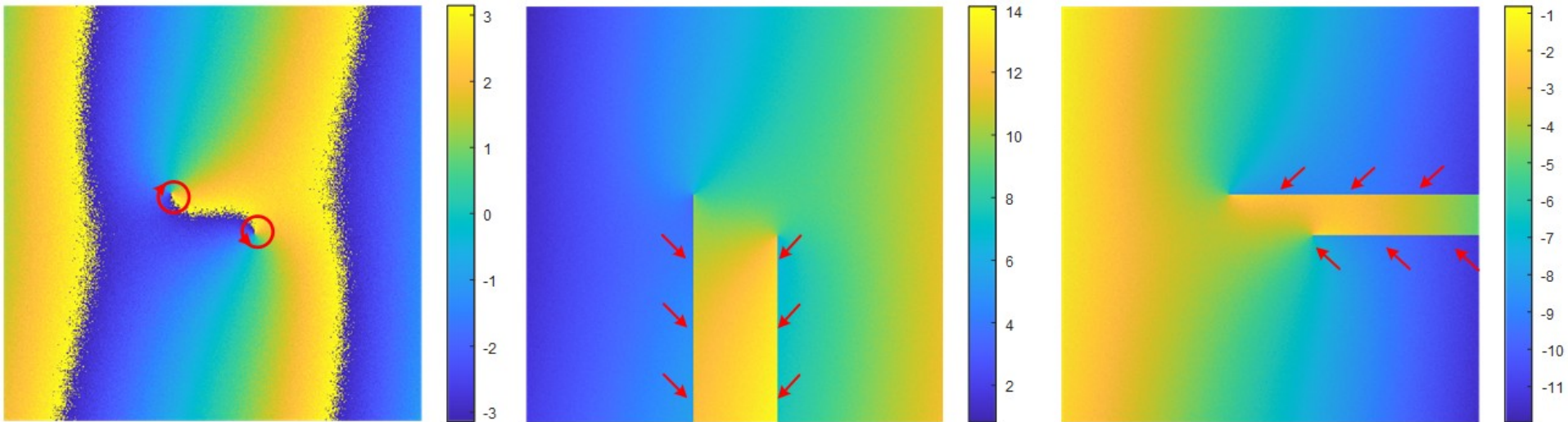
Phase discontinuity and discontinuity points





# 9 PHASE UNWRAPPING ERRORS

In the case of discontinuous phase, the unwrapping process will produce errors (artifacts). The absolute phase in this situation cannot be unambiguously restored.



**A**

The initial discontinuous interferogram

(the red circles show the phase growth)

**B**

Unwrapping in the vertical direction

(arrows show the boundaries of the artifact expressed

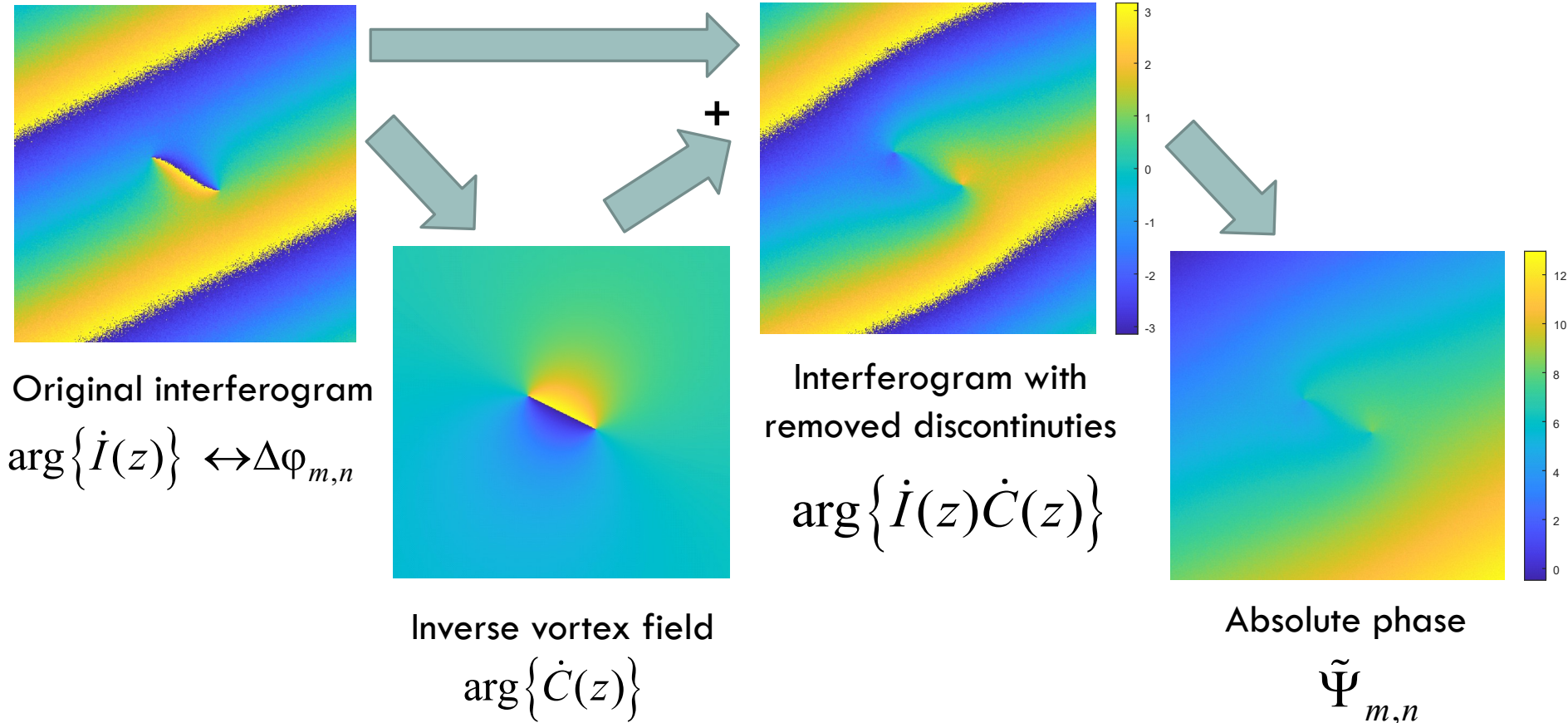
in extra interference fringe)

**C**

Unwrapping in the horizontal direction

# INVERSE VORTEX FIELD METHOD FOR PHASE UNWRAPPING

The algorithm, which resolves the phase discontinuities, uses the model of unit vortex (slide 8) with the opposite sign. After summing the interferogram and inversed unit vortices for all points, the continuous interferogram is obtained. Such interferogram can be transformed to the absolute phase without errors and artifacts.



# 11 INVERSE VORTEX FIELD METHOD: THE ALGORITHM

The algorithm for such phase unwrapping is realized using unconditional loop for passing all discontinuity points on the interferogram with forming the unit vortices and their complex multiplication to the interferogram.

1. Initialize of input data and parameters: interferogram and empty inverse vortex field .
2. Detect singular points where the residue function (phase difference on an elementary closed contour, [see slide 9](#)) is nonzero  
Calculate of the total number of singular points. If total number is zero, then go to step 4.
3. For each singular point:
  - calculate the fragment of the elemental vortex field centered at point
  - multiply the vortex field by the residue function value and add to the inverse vortex field ([see slide 10](#))

Compu  $\dot{C}_*(z) = \exp \left( j \cdot \arg \left\{ \frac{(z - z_{p1})^{q_{p1}} (z - z_{p2})^{q_{p2}} \dots (z - z_{p\mu})^{q_{p\mu}}}{(z - z_{01})^{q_{01}} (z - z_{02})^{q_{02}} \dots (z - z_{0u})^{q_{0u}}} \right\} \right)$  return to step 2.

4. Conve
5. Compute the absolute phase.

# PARALLEL IMPLEMENTATION

12

**This algorithm can be easily parallelized, unlike the most other unwrapping algorithms. It is realized on multicore processor and the GPU.**

## **Multithreaded CPU code:**

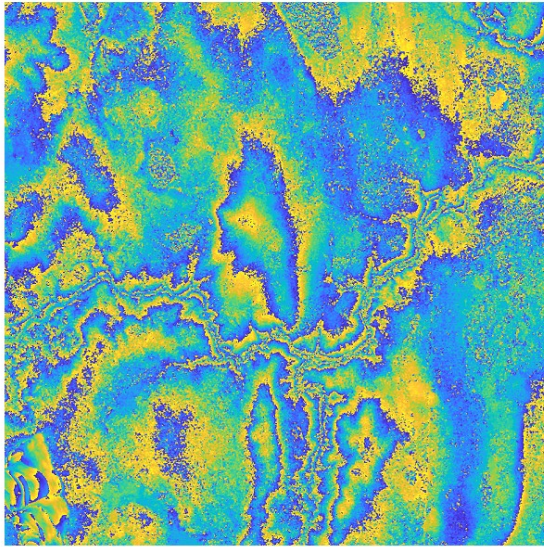
- The elemental vortex field twice the size of the interferogram is precalculated and fragments (centered at singular points) are obtained by computing the relative pointers;
- The outer loop iterated over rows of images and is distributed to OpenMP threads;
- The middle loop iterates over the list of singular points;
- The inner loop iterates by columns and is vectorized and tiled to blocks to utilize the cache and vector instructions.

## **GPU code:**

- Each CUDA thread processes a single image element;
- Inverse vortex field and interferogram fragments are preloaded into the local memory.

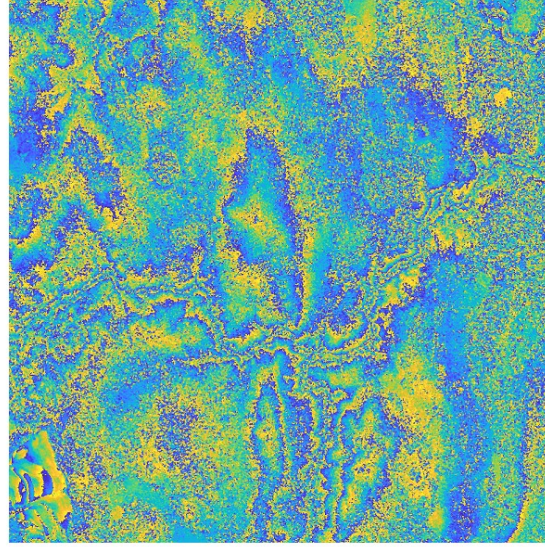
# 13 NUMERICAL EXPERIMENTS

Interferograms by the ALOS PALSAR radar, area of [Asbest city, Sverdlovsk Region, Russia](#)



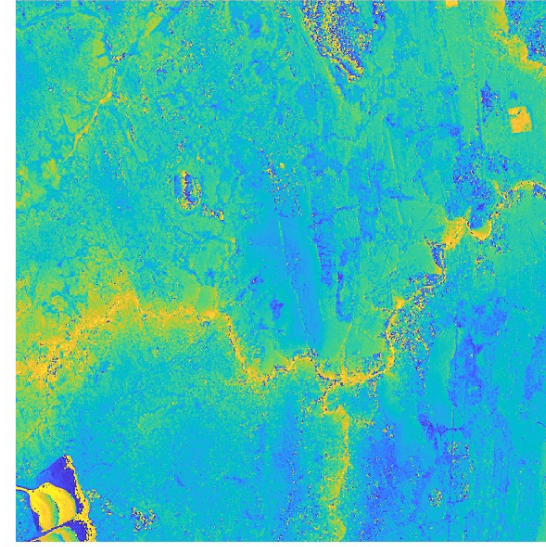
A

Scene 1: HH-polarization,  
m



B

Scene 2: HV-polarization,  
m



C

Scene 3: HH-polarization,  
m

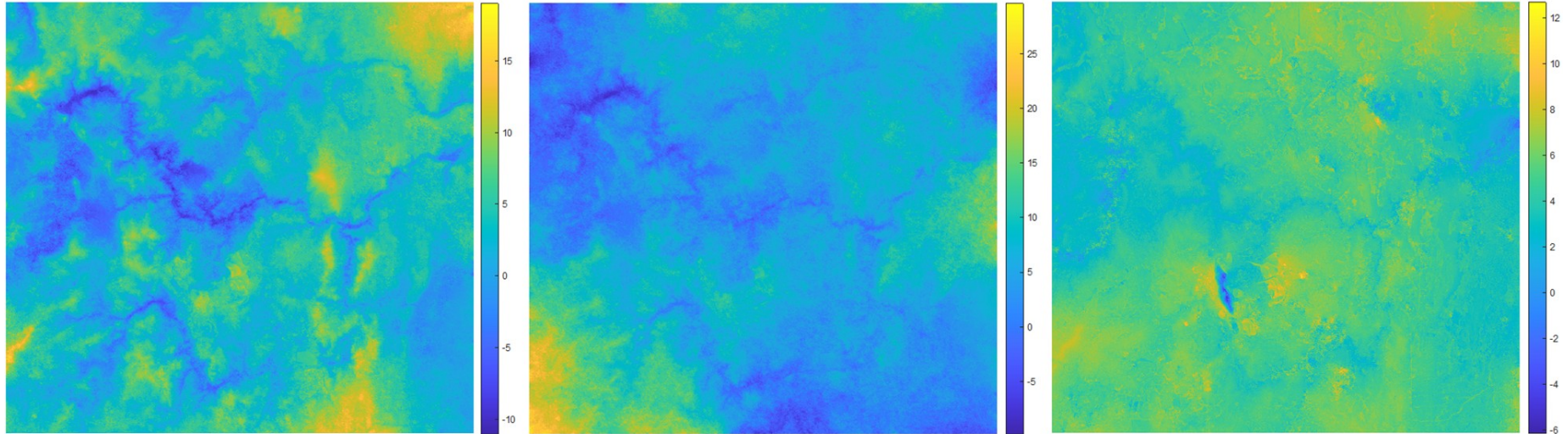


Initial scene size was samples for Scenes 1 and 2, and for Scene 3. The interferograms were resized to .

The reference terrain heights are represented as a set of at least points from a topographic map with a vertical accuracy of 0.5 m.

The reference heights were recalculated to the reference absolute phases

# NUMERICAL EXPERIMENTS: UNWRAPPED SCENES



A

Scene 1

=5.8%

Absolute phase error

B

Scene 2

=11.1%

C

Scene 3

=14.0%

Relative phase error

Here,  $\epsilon$  is the absolute phase error,  $\delta$  is the relative phase error,  $N$  is the number of reference points,  $\phi_i$  are the absolute phases,  $\phi_r$  are the reference absolute phases,  $\sigma_r$  is the reference absolute phase standard deviation.

## NUMERICAL EXPERIMENTS: COMPUTING TIME

Table contains the computing times of the CPU and GPU code for the scene 2. It contains the time for the CPU code for various numbers of OpenMP threads and time for the GPU code. The speedup coefficients  $S$ , where  $T_n$  is the computing time for  $n$  OpenMP threads, and  $T_1$  is the time for serial (single-threaded) code. For the GPU, the speedup coefficient is  $S_{GPU}$ , where  $T_{GPU}$  is the time obtained on the GPU.

Device and number of threads	Computing time , seconds	Speedup	Efficiency
AMD Ryzen 7500H 1 thread	24.4	—	—
2 threads	12.2	2.0	1.0
4 threads	6.3	3.9	0.97
8 threads	4.3	5.7	0.7
NVIDIA GeForce RTX 3070 Mobile	2.4	10.1	—

# NUMERICAL EXPERIMENTS: COMPUTING TIME

Table contains the computing times of the CPU code for the test interferogram of size and Intel i9-12900k (8C + 8c) processor.

Number of OpenMP threads	Cores configuration	Computing time , seconds	Speedup	Efficiency
1	1C	390	—	—
2	2C	205	1.90	0.95
4	4C	98	3.98	0.99
8	8C	49	7.96	0.99
16	8C HT	46	8.48	
16	8C HT AVX-512	41	9.51	
16	8C+8c	37	10.54	
24	8C HT + 8c	35	11.14	



# NUMERICAL EXPERIMENTS: COMPUTING TIME

Table contains the computing times of the CPU code for the test interferogram of size and Xeon Gold 6254 (18C) processors.

Number of OpenMP threads	Cores configuration	Computing time , seconds	Speedup	Efficiency
1	1C	730		
2	2C	365	2	1
4	4C	189	3.86	0.97
8	8C	92	7.93	0.99
12	12C	61	11.97	0.99
16	16C	47	15.53	0.97
18	18C	42	17.38	0.96
36	2 x 18C	22	33.18	0.92
18	18C AVX2	60		

# NUMERICAL EXPERIMENTS: SCALING

Table contains the computing times of the CPU code for the test interferograms of varying size and Intel i9-12900k (8C + 8c) processor.

Image size	Computing time	Scaling factor
1024x1024	2.6 seconds	
2048x2048	38 seconds	15.21
4096x4096	9.7 minutes	16.57
8192x8192	2 hours 42 minutes	16.84
10973x11001	11 hours	

# CONCLUSIONS

- A parallel codes for central and graphics processors based on OpenMP and CUDA technologies has been developed for solving the interferometric phase unwrapping problem in radar systems for remote sensing of the Earth.
- Using real SAR images, numerical experiments were carried out to compare the performance of the developed GPU code with the code previously developed for multicore CPUs.
- For CPU code, **11-fold speedup** was achieved for 16-core processor. **33-fold speedup** was achieved for 18-core processors.
- **Two-fold speedup** was achieved for the GPU code in comparison with the multithreaded CPU code (**8 threads**).
- The **relative error** for the best case is about **10%**, which is sufficient for various topographic applications.

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**Thanks for your attention!**

